Extracting Lumped-Element Parameters from a Finite Element Three-Phase Transformer Model – A Practical Approach –

A. Sauseng*, M. Kaltenbacher*, and K. Roppert*
* Institute of Fundamentals and Theory in Electrical Engineering, Graz University of Technology, Graz, Austria
alexander.sauseng@tugraz.at

This contribution focuses on a lumped-element parameter extraction of a finite-element three-phase transformer model. The two-dimensional transformer model is simulated using the A-formulation in the finite-element solver OPENCFS. The lumped-element model is based on modified nodal analysis and utilizes *Hopkinson*'s analogy to derive a magnetic equivalent circuit. The core in both approaches is described using a saturation function. The parameter extraction of the finite-element model for the lumped element model is discussed practically, and the simulation results are compared.

***keywords* – finite-element model, lumped-element model, three-phase transformer, nonlinear simulation**

I. Introduction

Simulating transformers is essential during their design phase, power transient analyses, and grid simulations. The finite-element (FE) method delivers accurate results with the main disadvantage of extensive computational time. In contrast, lumped-element (LE) models yield reasonable solutions in a fraction of the computational time of a FE model. They are thus more appropriate in grid or distribution line simulations where transformer models are embedded.

This contribution proposes a practical parameter extraction method from a two-dimensional three-phase transformer model, which is simulated using the A-formulation in the FE solver OPENCFS [1]. A voltage excitation with a serial resistance is applied. The LE model is based on *Hopkinson*'s analogy to derive a magnetic equivalent circuit (MEC) [2]. The core is depicted by flux tubes, representing homogeneous regions in the steel sheets. The steel sheets are modelled in both the FE and the LE model by a nonlinear saturation function. Since the crucial point in the LE model is the choice of the stray parameters, their influences are discussed, and a practical approach to extracting these parameters from the FE solution is proposed. Finally, the simulation results of the primary currents in the no-load and the short-circuit case are compared, and the computational time is discussed.

II. Finite-Element Transformer Model

The FE transformer model is meshed using linear triangular elements with an approximate length of $20\,\mathrm{mm}$. The geometric dimensions of the transformer are listed in Table I, and the computational grid for the FE simulation is depicted in Figure 1.

transformer width	$1\,\mathrm{m}$
transformer height	$0.5\,\mathrm{m}$
core thickness	$50\,\mathrm{mm}$
coil width	$30\,\mathrm{mm}$
coil height	$390\,\mathrm{mm}$

TABLE I: Geometric transformer dimensions

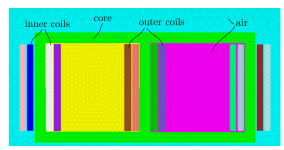


Fig. 1: Computational grid of the transformer

III. Lumped-Element Transformer Model

The LE model is based on *Hopkinson*'s analogy and models the core using a MEC. The LE transformer model, consisting of its electric ($a = \{1,3,5\}$ and $b = \{2,4,6\}$) and magnetic domain, is depicted in Figure 2 and Figure 3, respectively. The interconnection is enabled using controlled sources (◇ with $\frac{\mathrm{d}\phi}{\mathrm{d}t}$ the time derivative of the magnetic flux through the coil, ◇ with iN the magnetomotive force with N the number of winding turns and i the current through the coil). R_a and R_b represent the coil's resistances.

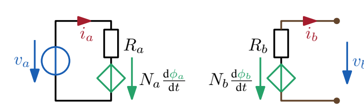


Fig. 2: Electric domain of the LE transformer model

The geometric parameters of a flux tube in the core result in

$$\mathcal{R} = \frac{l_{\text{mean}}}{A_{\text{core}}\mu}, \tag{1}$$

with l_{mean} the mean magnetic path length, A_{core} the core cross-section, and μ the nonlinear magnetic permeability. The individual regions for the flux tubes and the path lengths are depicted in Figure 4 with the upper and lower yokes in Figure 3 combined due to symmetry (i.e. twice the yoke length).

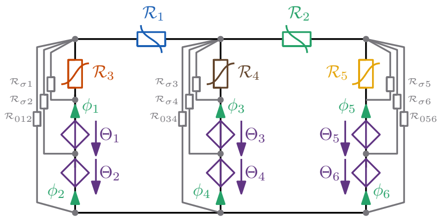


Fig. 3: Magnetic domain of the LE transformer model

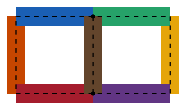
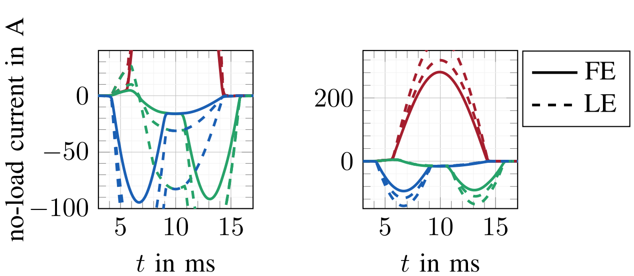


Fig. 4: Individual regions of the flux tubes and magnetic path lengths

IV. Determination of Stray Reluctances

The missing elements of the LE model in Figure 3 are the stray reluctances between the core and the inner windings ($\mathcal{R}_{\sigma,1}, \mathcal{R}_{\sigma,3}, \mathcal{R}_{\sigma,5}$), the stray reluctances between the inner and the outer windings ($\mathcal{R}_{\sigma,2}, \mathcal{R}_{\sigma,4}, \mathcal{R}_{\sigma,6}$) and the zero sequence relctances ($\mathcal{R}_{012}, \mathcal{R}_{034}, \mathcal{R}_{056}$). The proposed determination is to simulate the no-load and the short-circuit case and to manually tune these reluctances to the primary currents of the FE simulation. Since no tank walls and outer regions are modelled, $\mathcal{R}_{\sigma,1,3,5} =: \mathcal{R}_{\text{inner}}$, $\mathcal{R}_{\sigma,2,4,6} =: \mathcal{R}_{\text{outer}}$, and $\mathcal{R}_{012,034,056} =: \mathcal{R}_0$ holds.

First, the no-load case is simulated using the FE method. Since the core is non-saturated, the magnetic field closes mostly through the high permeability core and the zero sequence path in the air [3]. Thus, the stray path between the two coils $\mathcal{R}_{\text{outer}}$ is first neglected. We notice that $\mathcal{R}_0$ highly influences the shape of the primary current during zero-crossing, whereas $\mathcal{R}_{\text{inner}}$ highly influences solely the peak amplitude of the primary current. Thus, $\mathcal{R}_0$ is tuned first to yield the correct zero-crossing with $\mathcal{R}_{\text{inner}} = \infty$. Then, $\mathcal{R}_{\text{inner}}$ is manually decreased until the primary current shows the correct amplitude. This tuning is illustrated in Figure 5 for the no-load current (solid line: FE result; dashed lines: tuning the LE model).



(a) Zero sequence influence for $\mathcal{R}_{\text{inner}} = \infty$

(b) Inner stray path influence

Fig. 5: No-load tuning

The remaining quantity $\mathcal{R}_{\text{outer}}$ can be determined in the short-circuit case. The core is now saturated, allowing the inner coil's generated flux to close around $\mathcal{R}_{\text{outer}}$. The tuning is illustrated in Figure 6 for the short-circuit current (solid line: FE result; dashed lines: tuning the LE model).

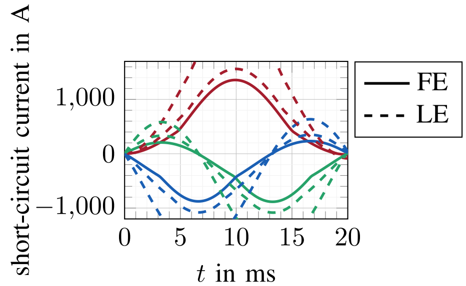
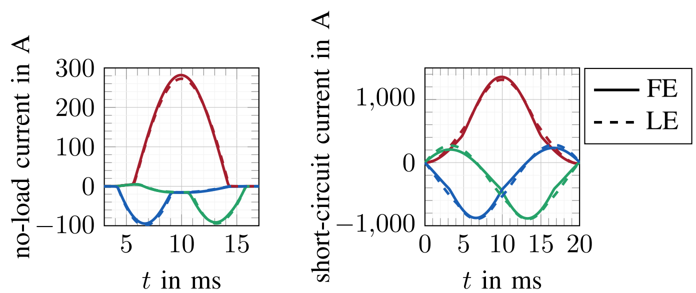


Fig. 6: Short-circuit tuning

V. Comparison and Conclusion

After extracting the mean flux path and the cross-sections from Figure 4 as well as $\mathcal{R}_0$, $\mathcal{R}_{\text{inner}}$, and $\mathcal{R}_{\text{outer}}$, the results of primary currents of the FE and LE model are depicted in Figure 7 (solid line: FE result; dashed lines: LE).



(a) No-load current

(b) Short-circuit current

Fig. 7: Comparison of the primary current between FE and LE model

The time step size has to be small enough (at least 100 time steps per 50 Hz-excitation period) to keep the discretization error small enough. The computational time of the FE model is in the minute's range, whereas the LE model needs under 200 ms to yield one period of the transformer model. The full contribution discusses the parameter tuning in more detail, the transformer's inrush to the steady state, and the material law (saturation function) in more detail.

References

[1] Kaltenbacher M. et al. *openCFS - open source finite element software for multi-physical simulation, ver. 2024S.* URL: https://gitlab.com/openCFS/cfs.

[2] P. Blanken and J. Van Vlerken. "Modeling of electromagnetic systems". In: *IEEE Transactions on Magnetics* 27.6 (1991), pp. 4509–4515. DOI: 10.1109/20.278672.

[3] S. Zirka et al. "Topology-Correct Reversible Transformer Model". In: *IEEE Transactions on Power Delivery* 27.4 (Oct. 2012), pp. 2037–2045. ISSN: 1937-4208. DOI: 10.1109/tpwrd.2012.2205275.